Percolation of localization in WSNs

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Abstract
Many localization mechanisms have been proposed for use in wireless sensor networks (WSN). In this paper, we investigate propagation properties of localization through simple trilateration across the network. We show that location information propagates following the *theory of percolation*. We empirically study the relationship between convergence speed of this process vs. node and network parameters: packet inter-arrival time and network degree.

1. Introduction

Today wireless sensor networks (WSNs) are being regarded as a promising option for data collection & delivery to a large number simple nodes connected with limited energy radio links. In order to improve data collection, modern protocols use node location. Location discovery in WSNs has been a hot topic since it is vital in environmental monitoring, huge machine surveillance and rescue operations like fire brigade intervention or natural disaster recovery.

One possible way to localize sensor nodes is to use the commonly available GPS [1] but this option can't be deployed largely because of its considerable power consumption and outdoor-only availability. Usual solution is to equip a limited number of nodes deployed outside a building with GPS receivers or to configure x,y coordinates of some specific nodes while deploying a WSN in-doors. These nodes called anchor nodes (ANs) serve as reference for other nodes and help them in their location discovery. These ANs possess the same capabilities including the same transmission range as of ordinary sensor nodes and only help their direct neighbors in getting localized. The new location aware sensor nodes then help their own neighbors and the process is iterated across the whole network to localize as much nodes as possible.

Many localization algorithms have been proposed and evaluated with respect to their accuracy, precision, calculation complexity or hardware constraints [2][3][4][9][10][12].

In this paper, we study the way in which unlocalized sensor nodes (UNs) become localized nodes (LNs) as the localization wave progresses in the network. We show that this flow of location information resembles the flow of a fluid across a random medium. This concept has been mathematically modeled by the *theory of percolation*. [5][6][7] have studied some global wireless network properties and have established their relationship with percolation theory but, to our knowledge, we are the first ones to evaluate the localization phenomenon in terms of its propagation as a percolating process. We show that the convergence speed of this process is linear and isotropic across the network. We then present the relationship between this speed with respect to node and network parameters like packet inter-arrival time and network degree.

Rest of the paper is organized as follows: next section highlights the related work in this domain. It is followed by the description of our approach. Section 4 talks about the simulation results and their relation with the percolation theory. Section 5 covers the details of estimating the convergence threshold and behavior of convergence speed by varying the packet inter-arrival time. Section 6 concludes our paper and presents possible directions for future work.

2. Related work and problem statement

Literature related to wireless sensor network localization consists of schemes either using (GPS-based) reference/anchor nodes [3][4] or approaches adhering to (GPS-less) anchor-free local coordinate system of the sensor network [9][10]. GPS-based schemes are classified into two main categories: fine grained and coarse grained localization schemes.

The fine-grained schemes [3][4] use perfect distance measurements among UNs and the ANs to produce accurate position estimates. Coarse-grained schemes on the other hand only give a rough estimate of a nodes location [11].

The first phase of node localization called *ranging phase* is based on communication among nodes. For this, it can make use of various available distance estimation techniques like ToA, TDoA or RSSI. Physical medium for ranging phase can use RF, acoustic or UWB signals, each of which has its pros and cons.

After the estimation of this distance, the second phase of node localization performs the computations on data (AN coordinates and distance) received in the first phase. There are many strategies like tri-lateration [1], atomic multi-lateration [3], min-max [4] for deriving each node's position from the information collected in the first phase.

Third phase of node localization involves refinement of initial estimates obtained in the computation phase. [3][4][12] discuss them in detail.

Anchor node placement

The performance of any localization scheme depends to a large extent on the way ANs are placed in the network. Popular location discovery schemes [2][3][4][12] have studied the effect of AN percentage and their placement on the proposed approaches. In all of these approaches, authors assume that UNs know enough reference points to evaluate their positions. These reference points can either be the *real* ANs (configured with their coordinates manually) if they are in the communication range of UNs (see fig 1a) or they can be common LNs closer to ANs in the case of multi-hop localization algorithms (see fig 1b). In both cases, a small percentage of network nodes are considered as ANs and are randomly deployed in the network. The benefit of this deployment is that it limits the mean number of hops from any UN to the nearest ANs and hence restricts the localization error propagation.

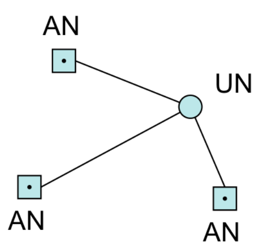

Fig 1a: trilateration is possible when at least three AN are in 1-hop distance from UN

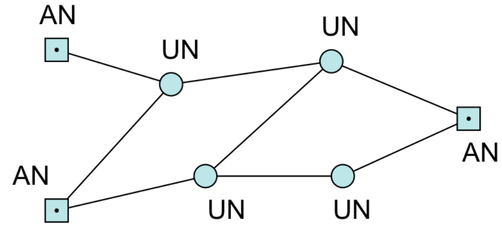

Fig 1b: basic trilateration is not suitable when no UN is a direct neighbor of three different ANs

In the case of localization through trilateration, placing the ANs randomly (figure 1b) in a network is not appropriate because no single UN is in view of 3 distinct ANs and ranging over multiple hops will yield highly erroneous distance estimates. Thus, a much better alternative is to place the three ANs in the center of the network non-collinearly within each other's transmission range as shown in fig 2a and 2b. We call this AN group an *AN nucleus*.

Localization activity starts from the *AN nucleus* with UNs in its vicinity. Once localized these UNs serve as LNs for the next hop UNs (figure 2b).

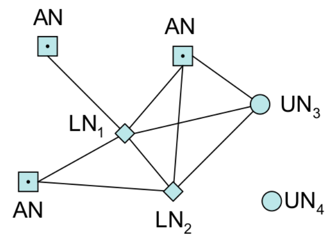

Fig 2a: LN1 is direct neighbor of 3 ANs. It performs trilateration and becomes localized. LN2 is localized with support of two ANs and LN1. UN3 will be localized with the help of AN, LN1 and LN2.

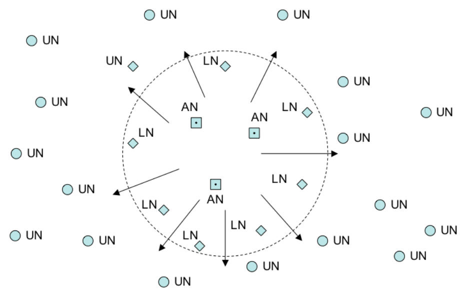

Fig 2b : *AN nucleus* starts the trilateration process which iterates and spreads across the network

With such a localization scheme, we are interested in finding out:

1. What is the convergence condition for this process?
2. What is the nature of convergence speed of this process (linear/logarithmic/exponential)?
3. How the convergence speed of this process is related to network parameters like each node's packet inter-arrival time, network degree?

3. Implementation of trilateration process

We have implemented the trilateration process in a very basic way in order to avoid any complex protocols between nodes. The trilateration process starts by ANs broadcasting beacon messages (BM) with a random inter transmission time. These BM messages are received by the neighboring UNs. Upon receiving three different beacons from three distinct ANs, the UNs broadcasts query messages (QMs). For the sake of simplicity, let's consider a single UN broadcasting a QM. This message is received by the entire neighborhood including the three ANs. Each of these ANs then sends a unicast response message (RM) to the demanding UN (fig 3). AN's response messages bring the anchor node's x,y (we consider 2D) coordinates plus internal round trip delay *Δt* at the ANs. Each UN can now estimate its distance from each AN through the following formula:

$$D_i = c.\frac{t_2 - t_1 - \Delta t}{2} \qquad (1)$$

where c is the propagation speed of the medium used in ranging (either light or sound).

The UNs then use these three distances and sets of AN coordinates to estimate their own coordinates in 2D assuming accurate distance estimations. These UNs become LNs and iterates the same process by broadcasting their own BMs. Note that our approach can be considered as the basic form of [2] but we keep our work simple by not considering the issues arising from bad node geometry.

Although an iterative localization process results in accumulation of error in every step of the process [12], we assume that our approach gives a good location estimate for each new localized node. This assumption helps us to focus on information *"propagation"* across the network.

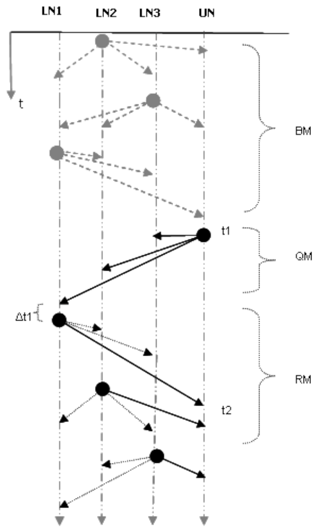

Figure 3: Message exchange during single trilateration

Assumptions

To keep our study simple, we assume an ideal propagation model in which every message transmitted on a wireless channel is received by the destination and it is not distorted by the interference from the environment. We also assume that the under lying MAC layer is pure CSMA. This old MAC protocol is not optimized for WSNs but it is a conservative assumption in our case because medium access load is always very low[1].
These assumptions simplify our approach and relieve us of the position refinement process that needs to be carried out at the end of node localization (see step 3 in section 2 above).

We further assume that our node localization process uses RF TOF 2-way ranging to estimate 1D distances between the *AN nucleus* and the UNs. It is an interesting technique since its pair wise roundtrip mechanism does not require clock synchronization. Also, the request response nature of this scheme subtracts the individual clock biases of the communicating nodes [15].

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[1] With 100 bit-long MAC messages, a transmission rate of 100kbps, η=15 and λ=5s, mean access load is 0.3% only.

4. Simulated network and results

We have used OPNET simulation tool [8] to study our approach.

We consider a sensor network of size L x L with *AN nucleus* in the center to limit the network border effects [14]. The N UNs placed randomly according to a uniform distribution.

For each node whether AN, LN or UN, the packet transmission is a Poisson process with an exponential inter-arrival time λ. Transmission range is the same for all nodes and is set to R meters considering a unit disk graph model.

Main parameters for our network are:
- Node density ρ

$$\rho = \frac{N}{L^2} \qquad (2)$$

- Network degree η (i.e. mean number of neighbors)

$$\eta = \rho\pi R^2 - 1 \qquad (3)$$

Critical network degree

To investigate our first question, we observed the relationship between the convergence speed of this process and mean network degree. In all of our simulations, we keep the node density constant to 0.015 m^{-2}. We alter the mean network degree by varying the transmission range of nodes and control the network size by varying the number of nodes.

Figure 4 illustrates curves for two network sizes. We observe sigmoid curves indicating a critical value of network degree for each network size. Below this critical value, only a small percentage of nodes is able to localize whereas above this critical value, the number of UNs that change to LN increase considerably.

Curves of Figure 4 are well known by percolation specialists. These "phase transition curves" mark the most important characteristic of a percolating system. We explain these curves by network connectivity.

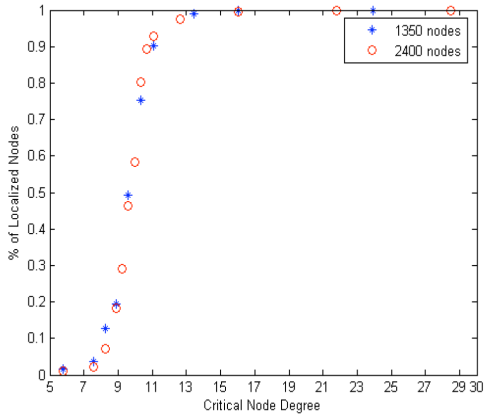

Fig 4: critical network degree to get a fully localized network

Under the critical node degree, the desired network connectivity is not enough. Location propagation wave can reach to a few nodes close to the *AN nucleus*. As the network degree increase, the cluster of LNs surrounding the *AN nucleus* increases in size but remains isolated, as with such a degree, the network does not have enough links to merge other clusters to the central cluster. When the network degree goes above the critical value, required connectivity becomes sufficient: now the wave can propagate throughout the network merging all clusters and localizing most of the nodes in the network.

Isotropic and linear propagation

Figures 5 show the convergence latency of UNs versus their Euclidean distance to the *AN nucleus*.

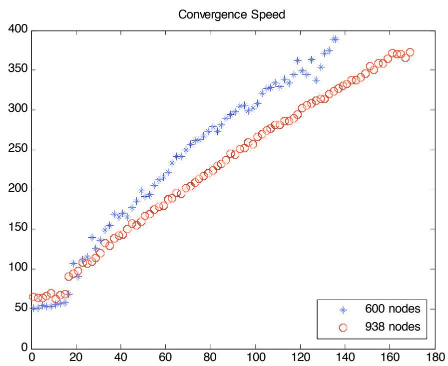

Fig 5: Convergence latency versus distance from AN nucleus (η = 14 for N=600, η=21 for N=938, λ=5s for both networks)

We see that the state change delay/speed of this process is linear with respect to distance from the *AN nucleus*, apart from the nodes in the direct neighborhood of ANs where propagation has no effect (i.e. they are in the *AN nucleus* vicinity).

Furthermore, detailed analysis of simulation results show that all nodes at a given distance in all directions of the *AN Nucleus* are localized at the same time. It leads us to conclude that the location dissemination wave propagates isotropically across the network.

These observations substantiate our conjecture (of figure 4) about the percolating behavior of this localization approach.

5. Use of percolation theory

Observations reported in section 4 persuade us to think that we can use percolation theory to answer questions like:

- For a given network (N, R, λ, η) can we estimate the value of critical network degree above which a network will surely get localized?
- Can we evaluate the convergence speed of this process?

Both of these issues are related to the energy consumption of this localization approach: estimate of the critical network degree will help us in spending only the required amount of energy in message communication and convergence speed can help us in determining the energy spent in this process. Answers to these questions will facilitate the practical use of our approach on sensor nodes.

Evaluation of critical network degree

In our simulation study, we have shown that location information percolates throughout the entire network when network degree is above a critical threshold value. This threshold is clearly related to the network connectivity. Our approach needs more than the usual 1-connectivity to carry out location discovery. In fact, it requires a 3-connected network.

A natural question that arises here is: how many neighbors on average do we need to achieve this desired 3-connected network? In this regard [13] has derived a mathematical formula for determining the mean number of neighbors needed for 1-connected network which is:

$$\eta = C(\log N) \qquad (4)$$

where C determines the upper and lower bounds for the network degree but is not fully defined up to now. We have used this formula to estimate the required node degree for 3-connectivity and have compared this value with the values indicated by our simulation results. Table 1 contains a comparison of the outcomes of equation 4 and the η observed in figure 4:

Number of Nodes	(C=1.0) Observed η	(C=1.0) Calculated η
1350	9.6	9.39
2400	9.8	10.1

Table 2: Critical network degree: simulation and formula (4) outcomes

The critical η values obtained in the two ways are quite close but further work needs to be carried out to achieve concrete results.

Convergence speed

We have analyzed the convergence speed of our approach as a function of packet inter-arrival time and network degree. Figure 6 and table 2 show these relationships.

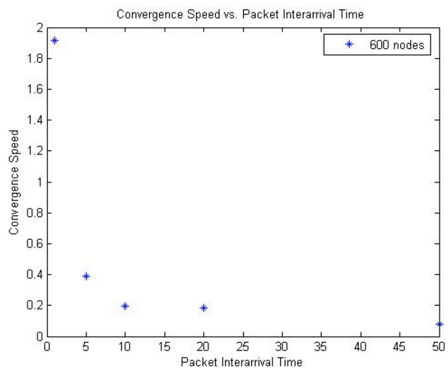

Fig 6: convergence speed as a function of inter-arrival time (η=14)

Range (m)	22	28	35
Network degree	21,8	35,9	56,7
Convergence speed (m/s)	0,54	0,71	0,91

Table 2: convergence speed as a function of network degree (N=1350, λ=5s)

Clearly the convergence speed of this process depends on nodes parameter (λ) and network parameter (η). Supplementary studies in this direction can help us in forecasting properties of this WSN localization scheme before its practical deployment.

6. Conclusion and Future perspective

In this paper, we have studied a simple and basic sensor node localization approach using trilateration with two-way ranging. We have empirically shown the existence of a critical network degree above which localization percolates in the entire network. We have also observed that the convergence speed of such process is set by key parameters of nodes and network.
This paper presents results with a set of only three ANs placed as a group with an ideal medium access and propagation environment. We plan to evaluate this approach in a realistic setup with augmented form of trilateration and additional ANs placed in the surrounding of *AN nucleus* to reduce error propagation.

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