Comparison of the Discriminatory Processor Sharing Policies

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1. Introduction

Discriminatory Processor Sharing (DPS) policy introduced by Kleinrock [5] is of a great interest in many application areas, including telecommunications, web applications and TCP flow modelling. Under the DPS policy the jobs priority is controlled by a vector of weights. Verifying the vector of weights, it is possible to control the service rates of the jobs and optimize system characteristics. The proper vector weights selection is an important and difficult task because of the system complexity. The previously achieved results on DPS model are of Kleinrock [5], Fayolle et al. [3]. Most of the results obtained for the DPS model were collected together in the survey paper of of Altman et al. [1]. The problem of weights selection in the DPS policy when the job size distributions are exponential was studied by Avrachenkov et al. in [2] and by Kim and Kim in [4]. In [4] it was shown that the DPS policy reduces the expected sojourn time in comparison with PS policy when the weights increase in the opposite order with the means of job classes. Also in [4] the authors formulate a conjecture about the monotonicity of the expected sojourn time of the DPS policy. The idea of conjecture is that comparing two DPS policies, one which has a weight vector closer to the optimal strict priority policy vector has smaller expected sojourn time. Using the method described in [4] we prove this conjecture with some restrictions on system parameters. The proofs can be found in the technical report [6].

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2. Main Definitions and Previous Results

We consider the DPS model with M job classes and a single server. Jobs of class $k = 1, \dots, M$ arrive with a Poisson process with rate λ_k and have required service-time distribution $F_k(x) = 1 - e^{-\mu_k x}$ with mean $1/\mu_k$. The load of the system is $\rho = \sum_{k=1}^M \rho_k$ and $\rho_k = \lambda_k/\mu_k$, $k = 1, \dots, M$. We consider that the system is stable, $\rho < 1$.
The state of the system is controlled by a vector of weights $g = (g_1, \dots, g_M)$, which denotes the priority for the job classes. If in the class k there are currently N_k jobs, then each job of class k is served with the rate equal to $g_j / \sum_{k=1}^M g_k N_k$, which depends on the number of jobs in each class.
Let $\overline{T}^{DPS}(g)$ be the expected sojourn time of the DPS system with weight g. Then $\overline{T}^{DPS}(g) = \sum_{k=1}^M \frac{\lambda_k}{\lambda} \overline{T}_k$, where $\overline{T}_k$ are expected sojourn times for classes $k = 1, \dots, M$ and can be found as a solution of the system of linear equations, see [3],

$$\overline{T}_k \left(1 - \sum_{j=1}^M \frac{\lambda_j g_j}{\mu_j g_j + \mu_k g_k} \right) - \\ - \sum_{j=1}^M \frac{\lambda_j g_j \overline{T}_j}{\mu_j g_j + \mu_k g_k} = \frac{1}{\mu_k}, \quad k = 1, \dots, M. \quad (1)$$

Let us notice that when all weights are equal, the DPS system is equivalent to the standard PS system.
One of the problems when studying DPS is to minimize the expected sojourn time $\overline{T}^{DPS}(g)$ with some weight selection. This is a general problem and to simplify it the following subcase is considered. To find a set G such that $\overline{T}^{DPS}(g^*) \leq \overline{T}^{PS}$, $\forall g^* \in G$. For the exponential job size distributions it was shown in [4] that if the means of the job classes are enumerated such as $\mu_1 \geq \mu_2 \geq \dots \geq \mu_M$, then

$$G = \{g | \; g_1 \geq g_2 \geq \dots \geq g_M\}. \quad (2)$$

Using the approach of [4] we solve more general problem about $\overline{T}^{DPS}(g)$ monotonicity according to the weight vector selection, which we formulate in the following section as Theorem 1.

3. Expected sojourn time monotonicity

Theorem 1 *Let the job size distribution for every class be exponential with mean μ_i, $i = 1, \dots, M$ and we enumerate them in the following way $\mu_1 \geq \mu_2 \geq \dots \geq \mu_M$. Let us consider two different weight policies for the DPS system, which we denote as α and β.*

Let $\alpha, \beta \in G$. The expected sojourn time of the DPS policies with weight vectors α and β satisfies

$$\overline{T}^{DPS}(\alpha) \leq \overline{T}^{DPS}(\beta), \qquad (3)$$

if the weights α and β are such that :

$$\frac{\alpha_{i+1}}{\alpha_i} \leq \frac{\beta_{i+1}}{\beta_i}, \quad i = 1, \ldots, M-1, \qquad (4)$$

and the following restriction is satisfied :

$$\frac{\mu_{j+1}}{\mu_j} \leq 1 - \rho, \qquad (5)$$

for every $j = 1, \ldots, M$.

Remark 2 *If for some classes j and $j+1$ condition (5) is not satisfied, then in practice, by choosing the weights of these classes to be equal, we can still use Theorem 1. Namely, for classes such as $\frac{\mu_{j+1}}{\mu_j} > 1 - \rho$, we suggest to set $\alpha_{j+1} = \alpha_j$ and $\beta_{j+1} = \beta_j$.*

Remark 3 *Theorem 1 shows that the expected sojourn time $\overline{T}^{DPS}(g)$ is monotonous according to the selection of weight vector g. The closer is the weight vector to the optimal strict priority policy, or μ-rule, the smaller is the expected sojourn time. This is shown by the condition (4), which shows that vector α is closer to the optimal strict priority policy then vector β. Theorem 1 is proved with restriction (5). This restriction is a sufficient and not a necessary condition on system parameters. It shows that the means of the job classes have to be quite different from each other. This restriction can be overcome, giving the same weights to the job classes, which mean values are similar. Condition (5) becomes less strict as the system becomes less loaded.*

The proof of Theorem 1 can be found in the technical report [6].

4. Numerical results

Let us consider a DPS system with 3 classes. Let us consider the set of normalized weights vectors $g(x) = (g_1(x), g_2(x), g_3(x))$, $\sum_{i=1}^{3} g_i(x) = 1$, $g_i(x) = x^{-i}/(\sum_{i=1}^{3} x^{-i})$, $x > 1$. Every point $x > 1$ denotes a weight vector. Vectors $g(x), g(y)$ satisfy property (4), namely $g_{i+1}(x)/g_i(x) \leq g_{i+1}(y)/g_i(y)$, $i = 1, 2$, $1 < y \leq x$. On Figure 1 we plot $\overline{T}^{DPS}(g(x))$. Here the parameters are : $\lambda_i = 1$, $i = 1, 2, 3$, $\mu_1 = 160$, $\mu_2 = 14$, $\mu_3 = 1.2$, then $\rho = 0.911$ and condition (5) is satisfied for three classes. One can see that $\overline{T}^{DPS}(g(x)) \leq \overline{T}^{DPS}(g(y))$, $1 < y \leq x$. On Figure 1 we plot the expected sojourn times $\overline{T}^{PS}$ for the PS policy and $\overline{T}^{st}$ for the optimal strict priority policy.

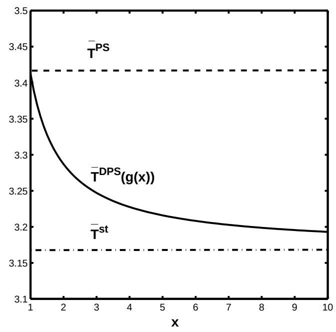

FIG. 1 – $\overline{T}^{DPS}(g(x))$, $\overline{T}^{PS}$, $\overline{T}^{st}$ functions.

5. Conclusion

We study the problem of minimization of the expected sojourn time for the DPS policy with weights selection when the job size distributions are exponential. With some restrictions on a system parameters we prove the monotonicity of the expected sojourn time of the DPS policy according to the weight vector selection. The restrictions on the system are such that the result is true for systems for which the values of the job size distribution means are very different from each other. The restriction can be overcome by setting the same weights for the classes, which have similar means.

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